Brightness luminous-efficiency functions for 2° and 10° fields

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CIE V_λ is not representative of luminous-efficiency function based on heterochromatic brightness matching. CIE Technical Committee 1.4 Vision (TC 1.4) presented a 2° brightness-matching luminous-efficiency function based on studies of a total of 31 observers to supplement the V_λ. In view of their importance to illuminating engineering and physiological optics, we analyzed the various conditions under which these seven studies were conducted. Data from three of the groups are considered inappropriate, and we revised the TC 1.4 brightness luminous-efficiency function based on the remaining 19 subjects. Data from 18 Japanese subjects, coming from five research groups, are added to the above subjects, and an averaged luminous-efficiency function is derived. The result does not appreciably differ from the revised TC 1.4 function and is considered to represent a brightness-matching standard luminous-efficiency function for a 2° field. A brightness luminous-efficiency function for a 10° field based on nine Japanese subjects is presented. It differs from the 2° function only at short wavelengths when the functions are normalized at 570 nm. A theoretical approach for using the brightness-matching luminous-efficiency function to assess the brightness of 2° broadband sources is introduced, and some numerical examples are given.

INTRODUCTION

Spectral luminous-efficiency functions may vary according to the methods used, for example, heterochromatic flicker photometry, minimally distinct border, visual acuity, increment threshold, and heterochromatic brightness matching. Fortunately, we have reason to believe that luminous-efficiency functions derived by using various methods may be grouped into two types,[1,2]

One function is relatively smooth with a maximum sensitivity at about 560 nm. This function, when used in the luminance equation [see Eq. (1)], satisfies the additivity assumption. The methods that yield this type of luminous-efficiency function are flicker photometry,[3–9] minimally distinct border,[2,10] and grating visual acuity.[11–13] Judd's modification of CIE V_λ is a good representation of this function.

The other function has a broader shape compared with that of CIE V_λ, often showing two peaks at about 540 and 600 nm, and is usually not additive, particularly when red and green parts of the spectrum are added to each other or when yellow and blue are added. The methods that yield this type of luminous-efficiency function are absolute threshold[14–20] and brightness matching.[2,14,21–27] No standard has been established for this function despite the importance of these visual tasks in our daily life. The TC 1.4, therefore, made an appeal for brightness-matching data so that a reliable, standard luminous-efficiency function could be derived.[28] So far, the committee has collected the available spectral luminous-efficiency data for 2° fields and tabulated the averaged values covering 400 through 700 nm.[29]

As chairman of the Subcommittee on Luminous Efficiency Functions for TC 1.4, one of the authors (Ikeda) made a similar appeal for more data directly to the national committees of the CIE. Although responses have been few, it is hoped that researchers will perform brightness-matching experiments and provide additional data for the purpose of standardization. Meanwhile, some Japanese researchers conducted experiments, and the national committee on luminous-efficiency functions that is affiliated with the Illuminating Engineering Institute of Japan summarized these data.[30]

The present paper includes these new Japanese data, thus improving the reliability of the existing data. A tentative luminous-efficiency function for 10° field is also presented.

FOUR VARIABLES

In our analysis of existing brightness-matching luminous-efficiency functions, we attempted to hold four variables approximately constant: stimulus size, retinal illuminance, number of wavelengths tested, and data from individual observers as opposed to aggregate data.

Stimulus Size

The visual angle subtended by the stimulus does not need to be exactly 2° or 10°. However, brightness-matching luminous-efficiency functions using point sources (approximately 2.3′) yield functions similar to heterochromatic flicker photometry with 2° fields.[31] Fields subtending visual angles greater than 22′ begin to approximate functions obtained with 2° fields. Therefore we set a lower limit of 1° and an upper limit of 3° in our analysis of data representative of 2° functions. When analyzing 10° functions, we used studies employing fields ranging from 7° to 12°.

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Table 1. Authors and Experimental Conditions Cited in This Paper[a]

Authors (Years)	Field	Level	Number of Subjects
Bedford and Wyszecki (1958)[22]	1°	50 td	4
Sperling (1958)[34]	2°	500 td	6
Sperling and Lewis* (1959)[14]	2°	500 td	3
Kinney* (1964)[35]	2°	0.1 fL	4
Wagner and Boynton (1972)[2]	1°40′	1–572 td	4
Guth and Lodge* (1973)[23]	45′	191 td	5
Comerford and Kaiser (1975)[24]	1°	150, 225, 400 td	5
Yaguchi and Ikeda (1980)[33]	2°	100 td	5
Uchikawa and Ikeda (1981)[36]	2°	76 td	3
Katori and Fuwa (1981)[37]	2°	50 td	6
Sagawa (1981)[38]	1°	80 td	1
Hasegawa (1982)[30]	2°	170 td	3

[a] Data of the authors whose names are followed by asterisks are deleted from the final result. 2° field.

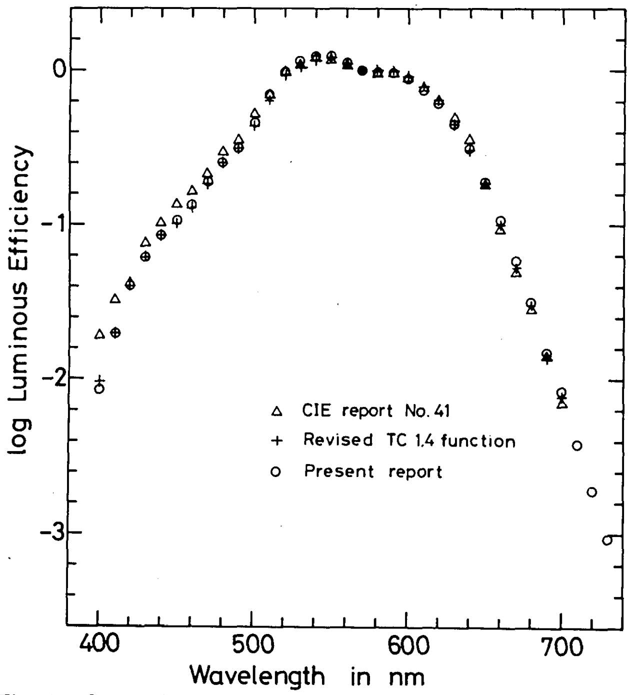

Fig. 1. Comparison of three luminous-efficiency functions for brightness in a 2° field. △, CIE TC 1.4 Report No. 41; +, revised TC 1.4 function, and ○, present paper. Normalized at 570 nm.

Retinal Illuminance

We restricted our analysis to studies employing about 100 td. Luminous-efficiency functions may change shape, even within the photopic range, as retinal illuminance is varied.[32,33] The value 100 td was chosen because modest deviations from this level do not cause shape changes in the luminous-efficiency functions.

Number of Wavelengths

As noted above, a brightness-matching luminous-efficiency function is not completely smooth. Therefore, in order to detect all the slope changes, it is necessary to sample a sufficiently large number of wavelengths throughout the visible spectrum. Measurements taken at a minimum of 20-nm intervals are desirable.

Individual Observers

We looked only at studies that presented data for their observers, as opposed to aggregate data. This was necessary so that the mean could be calculated directly from all individual data. The mean may vary depending on how it is calculated. In the present paper, we calculate the mean in logarithms, or the geometric mean, which differs from the commonly used arithmetic mean.

DATA FOR A 2° FIELD

Table 1 summarizes studies we surveyed for the 2°-field condition. The data of the first seven studies were reported in CIE Report No. 41[2,14,22–24,29,34,35] and resulted in the average shown by the triangles in Fig. 1. The total number of subjects was 31. The data of the bottom five studies employing 18 Japanese subjects were presented in the national committee report[30] on the luminous-efficiency functions.[33,36–38]

ANALYSIS AND RESULTS FOR 2° FIELDS

We first rechecked the data of the seven groups utilized in CIE Report No. 41 with respect to the four variables introduced above. The field size varies from 45′ to 2°, as seen in Table 1, but this range may be tolerable. Some retinal illuminances fail to satisfy the second principle of 100 td. In particular, Kinney's 0.1 fL is too low to ensure a normal photopic curve.[33] Wagner and Boynton's 1 td is considerably below 100 td, but fortunately this level was used only at 690 nm, and higher levels were employed for the other wavelengths. As to the number of wavelengths, Kinney used only seven. With regard to individual data, Guth and Lodge reported only the averaged values of five subjects. Sperling and Lewis's[14] three observers are included in Sperling's[34] six observers, and their data completely overlap each other.

We decided, therefore, to delete the data of Kinneys, Guth and Lodge, and Sperling and Lewis (marked with asterisks in Table 1) from further analyses. For the remaining 19 subjects, we calculated the geometric mean and obtained the result shown by crosses in Fig. 1. We call this result the revised TC 1.4 function. The value is normalized at 570 nm, as was done in CIE Report No. 41. The revised values do not appreciably differ from the original ones shown by triangles.

The five Japanese groups listed at the bottom of Table 1 meet the four criterion variables, and the data from 18 observers can be directly compared with those from the 19 observers of the revised TC 1.4 function. We put all 37 subjects together and calculated the geometric mean. The result is shown in Fig. 1 by open circles and is tabulated in Table 2. This average function is also presented separately in Fig. 2.

A few comments must be made in relation to the work of averaging individual data. Some subjects provided luminous efficiencies at wavelengths of 20-nm intervals. In this case, an intermediate value was determined by linear interpolation using the neighboring two wavelengths. No extrapolation was done to estimate data at extreme wavelengths. Therefore an abrupt change in the mean luminous efficiency might occur

Table 2. Brightness Luminous-Efficiency Function $V_b(\lambda)$ for 2° Field as a Final Result in This Paper[a]

λ (nm)	log $V_b(\lambda)$	ΔV_b	n
400	−2.07	−0.05	8
410	−1.71	−0.03	20
420	−1.40	+0.01	33
430	−1.22	0	36
440	−1.07	0	36
450	−0.98		37
460	−0.88		37
470	−0.73		37
480	−0.60		37
490	−0.50		37
500	−0.34		37
510	−0.15		37
520	−0.01		37
530	+0.06		37
540	+0.09		37
550	+0.09		37
560	+0.05		37
570	0.00		37
580	−0.01		37
590	−0.02		37
600	−0.06		37
610	−0.13		37
620	−0.22		37
630	−0.35		37
640	−0.51		37
650	−0.73		37
660	−0.97	+0.01	35
670	−1.23	+0.01	35
680	−1.50	+0.01	32
690	−1.83	+0.07	21
700	−2.08	+0.08	15
710	−2.42	+0.14	6
720	−2.72	+0.14	6
730	−3.03	+0.14	6

[a] Values are in logarithmic units. n denotes number of subjects, and ΔV_b is the adjusted amount for reduced subjects. Normalized at 570 nm.

at a wavelength at which the number of subjects was reduced at the extreme wavelengths if the simply averaged values were used. This artifact irregularity in the luminous-efficiency function may cause misinterpretation about the nature of the function and was avoided by plotting the function near the irregularity with a curve based only on the reduced number of subjects. For example, the geometric mean of 35 subjects at 660 nm is −0.98. The means of these same subjects are −0.74 and −0.52 at 650 and 640 nm, respectively, yielding differences of −0.01 and −0.01, respectively, when compared with the averages of all 37 subjects at these wavelengths. Therefore we add +0.01 to the geometric mean of the reduced number of subjects so that the final average is −0.97 at 660 nm. A similar modification was performed on all extreme wavelengths. The amounts of these adjustments are shown in column ΔV_b of Table 2. The straightforward averages of the original data can be obtained by subtracting the adjusted amount from the final values of Table 2, that is, (log V_b) − (ΔV_b).

The comparisons in Fig. 1 indicate that the data presented in this paper do not deviate significantly from the revised TC 1.4 function. The vertical lines in Fig. 3 illustrate the range of individual data of 37 observers. The mean function is indicated by open circles. The range is below the final values at extremely long wavelengths, but this is because the ranges are based on the original data before the adjustments were made at the spectral ends. The variance among 37 subjects is large, and yet the final function does not deviate much from the revised TC 1.4 function. This fact suggests that a further addition of individual data may not drastically change the present result. This does not mean of course that we do not need any further experimental results.

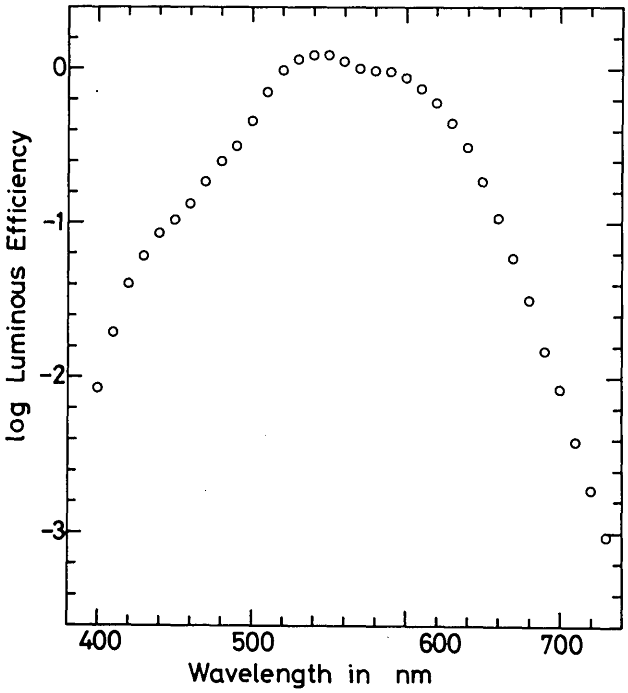

Fig. 2. Luminous-efficiency function for brightness based on 37 subjects in a 2° field. Normalized at 570 nm. Values are tabulated in Table 2.

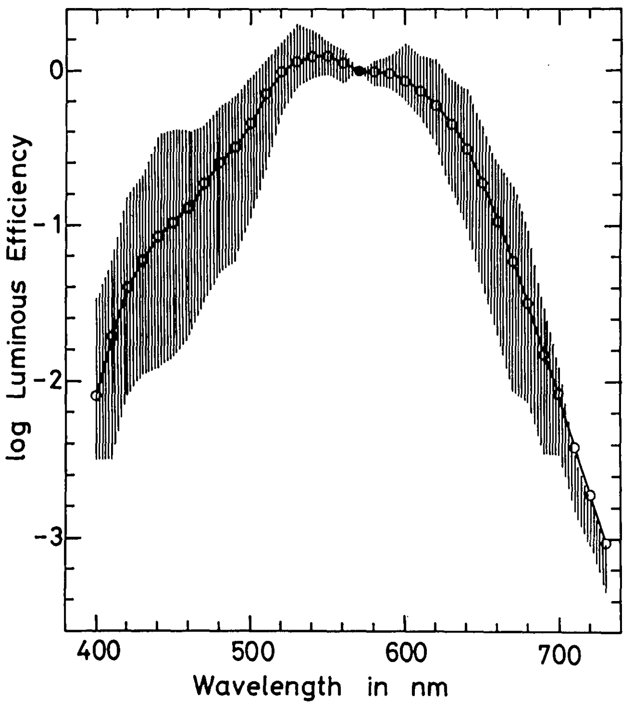

Fig. 3. Range of variation of individual data of 37 subjects. Circles represent the luminous-efficiency function of 2° derived in the present paper.

Table 3. Authors and Experimental Conditions for a 10° Field

Authors (Years)	Field (deg)	Level (td)	Number of Subjects
Ikeda and Shimozono (1981)[32]	10	100	4
Katori and Fuwa (1981)[37]	10	200	4
Sagawa (1981)[38]	8	50	1

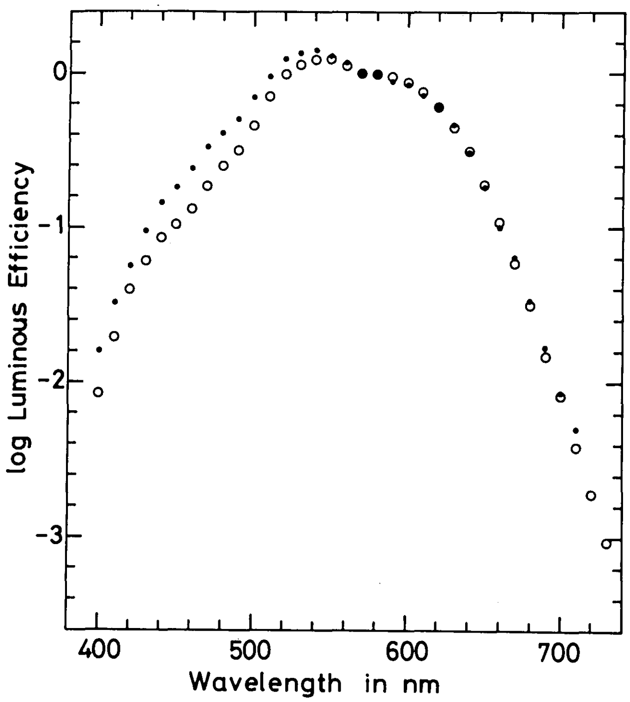

Fig. 4. Luminous-efficiency functions of 2° (open circles) and 10° (filled circles) for brightness.

TENTATIVE RESULTS FOR A 10° FIELD

Establishing a standard brightness luminous-efficiency function for a field of 10° is important because the visual tasks in which people engage usually involve large fields. We have not systematically surveyed the literature for research conducted with 10° fields. We would like to present preliminary results that were summarized in the report of the Japanese national committee on the luminous-efficiency function.[30]

Authors and experimental conditions are listed in Table 3.[32,37,38] There was a total of nine observers. The average is indicated by filled circles in Fig. 4, and the open circles represent 2° data that have been replotted from Fig. 2. The 10° function does not differ from the 2° function at wavelengths longer than 570 nm. Differences exhibited below 570 nm, however, are approximately 0.2 log unit.

DISCUSSION

The luminous-efficiency function is used in the CIE luminance equation in the form

$$L = K_m \int_\lambda L_{e\lambda} V(\lambda) \mathrm{d}\lambda, \tag{1}$$

where K_m is the maximum luminous efficacy with a value of 683 lm/W. Strictly speaking, the luminance L given by Eq. (1) is valid only when the luminous-efficiency function $V(\lambda)$ satisfies the additivity assumption required by the integral. It is obvious that, even if the luminous-efficiency function $V_b(\lambda)$ is established for the brightness perception as given by Fig. 2 or Table 2, we cannot simply replace $V(\lambda)$ in Eq. 1 with $V_b(\lambda)$ because of the additivity failure observed in the brightness matching. We have to find a formula different from Eq. (1).

It was first suggested by Ikeda[16] that the additivity failure in the increment-threshold experiment was due to the red–green opponency. Guth confirmed our result and also the additivity failure observed in the brightness-matching experiment by Tessier *et al.*[18,21,23] He proposed a visual model in which brightness perception was mediated by both the achromatic and the chromatic responses,[23] a notion that is now widely accepted.[10,39–42] An elegant formula was derived by Guth to calculate a psychophysical quantity called a vector luminance to represent the brightnesss perception. Yaguchi and Ikeda[26,43] extensively investigated the additivity failure of brightnesses and proposed a similar formula to calculate the psychophysical quantity L_b, corresponding to the brightness of a broadband or compound spectrum $L_{e\lambda}$. The formula is

$$\left[\int_\lambda \left(\frac{L_{e\lambda}}{L_b}\right) \bar{a}_\lambda \mathrm{d}\lambda\right]^2 + \left[\int_\lambda \left(\frac{L_{e\lambda}}{L_b}\right) \bar{c}_{1\lambda} \mathrm{d}\lambda\right]^{2p} + \left[\int_\lambda \left(\frac{L_{e\lambda}}{L_b}\right) \bar{c}_{2\lambda} \mathrm{d}\lambda\right]^{2q} = 1. \tag{2}$$

The first term represents the contribution of the achromatic channel, and $\bar{a}_\lambda$ is Judd's modification of $V(\lambda)$, or $\bar{y}'(\lambda)$. The second and third terms are contributions of the red-versus-green and the yellow-versus-blue opponent-color channels, respectively. $\bar{c}_{1\lambda}$ and $\bar{c}_{2\lambda}$ are spectral-response functions of these channels and are biphasic with respect to wavelength, as are the chromatic valences developed by Jameson and Hurvich.[44] p and q are constants smaller than unity assuming that $p = 0.64$ and $q = 0.36$. These values are necessary to explain the asymmetrical property of the additivity failure. If both p and q were unity, Eq. (2) would be essentially same as that proposed by Guth.

For equi-energy monochromatic light, we remove the integrals from Eq. (2) and put $L_{e\lambda} = 1$ for all wavelengths, namely,

$$\left(\frac{\bar{a}_\lambda}{L_b}\right)^2 + \left(\frac{\bar{c}_{1\lambda}}{L_b}\right)^{2p} + \left(\frac{\bar{c}_{2\lambda}}{L_b}\right)^{2q} = 1. \tag{3}$$

By solving this equation for $(1/L_b)$ for each wavelength, we can obtain the luminous-efficiency function for brightness, $V_b(\lambda)$, which must be equal to the function given in Fig. 2 or Table 2. We have not defined the response functions $\bar{c}_{1\lambda}$ and $\bar{c}_{2\lambda}$. We assume that $\bar{c}_{1\lambda}$ is the difference between red and green cone responses and that $\bar{c}_{2\lambda}$ is the difference between red-plus-green and blue cone responses. Further, we assume unique yellow and unique green at 577 and 500 nm, respectively, where $\bar{c}_{1\lambda}$ and $\bar{c}_{2\lambda}$ should become zero. These wavelengths were obtained based on responses of 13 normal subjects in various studies.[44–49] The standard deviations among the subjects were 6 and 7 nm in unique yellow and unique blue, respectively. The exact $\bar{c}_{1\lambda}$ and $\bar{c}_{2\lambda}$ responses were obtained by adjusting coefficients of cone responses so that the resultant $V_b(\lambda)$ became as nearly equal as possible to the experimentally determined luminous-efficiency function of Table 2.

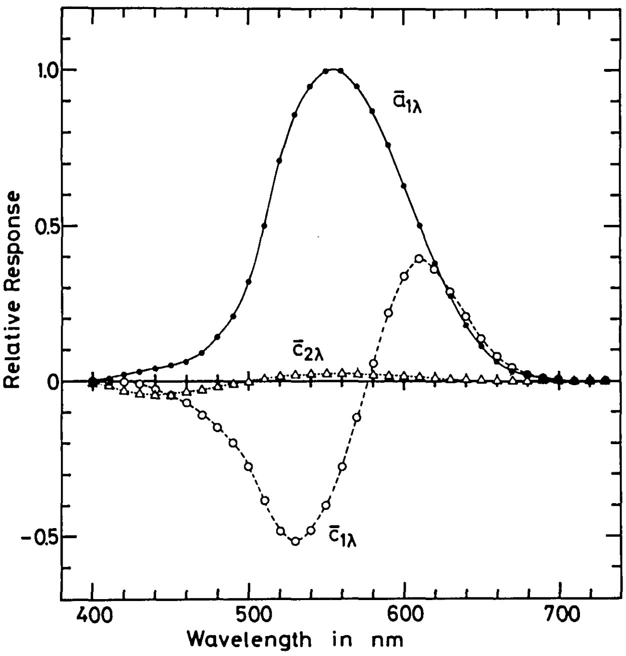

Fig. 5. Spectral responses of achromatic ($\bar{a}_\lambda$), red-versus-green ($\bar{c}_{1\lambda}$), and yellow-versus-blue ($\bar{c}_{2\lambda}$) chromatic opponent channels contributing to brightness perception.

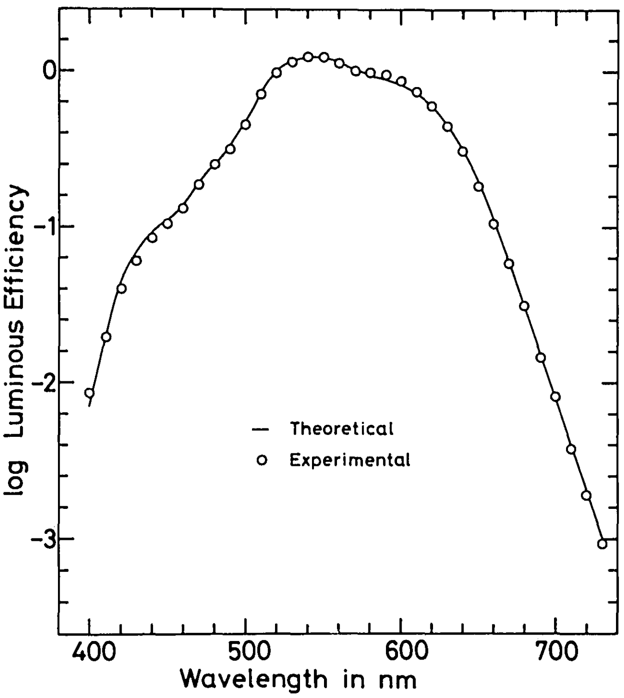

Fig. 6. Luminous-efficiency function of a 2° field for brightness derived experimentally (open circles) and theoretically (solid curve).

The final values of $\bar{a}_\lambda$, $\bar{c}_{1\lambda}$, and $\bar{c}_{2\lambda}$ are summarized as

$$\begin{pmatrix}\bar{a}\\ \bar{c}_{1\lambda}\\ \bar{c}_{2\lambda}\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 0.758 & -0.736 & -0.156\\ 0 & 0.024 & -0.029\end{pmatrix}\begin{pmatrix}\bar{x}'(\lambda)\\ \bar{y}'(\lambda)\\ \bar{z}'(\lambda)\end{pmatrix} = \begin{pmatrix}1 & 1 & 1\\ 1.496 & -3.388 & 0\\ 0.024 & 0.024 & -3.950\end{pmatrix}\begin{pmatrix}R_\lambda\\ G_\lambda\\ B_\lambda\end{pmatrix}. \quad (4)$$

$\bar{x}'(\lambda)$, $\bar{y}'(\lambda)$, and $\bar{z}'(\lambda)$ are the color-matching functions for the 2° field, and R_λ, G_λ, and B_λ are cone-response functions, both tabulated by Vos[50] in consideration of Judd's modification of $V(\lambda)$. Figure 5 is a plot of Eq. (4). The solid curve in Fig. 6 is the theoretical $V_b(\lambda)$ calculated by Eqs. (3) and (4) and is a close fit to the open circles. The luminous-efficiency function proposed in the present paper suggests the validity of Eqs. (2) and (3).

As a numerical example of calculating L_b, Eq. (2) was applied to two different spectral compositions shown by a thick solid curve (light A) and a thick dashed curve (light B) in Fig. 7. When luminances of both lights were equated to 100, the L_b value was 104 for light A and 173 for light B, giving a ratio of 1.66, which implies that light B is much brighter than light A. These L_b's are obtained because light B has energy only in the red part of spectrum, whereas light A has energy in both the green and red parts of spectrum, which cancel each other in the second term of Eq. (2).

This theoretical prediction was tested with real lights in an experiment in which two lights, A and B, were produced through colored filters in a 2° Maxwellian-view field. The two lights (curves A and B, Fig. 7) were first equated by flicker photometry to about 70 td. Then fields A and B were matched for brightness by direct comparison by adjusting the luminance of light B. The luminance ratio of light B between flicker photometry and direct comparison was obtained. Five subjects with normal color vision participated in the experiment and gave a mean ratio of 1.68, which is quite close to the prediction. The reduced radiance of light B for brightness matching is indicated by the thin dashed curve in Fig. 7. The brightness luminous-efficiency function shown in Fig. 2 should be useful in evaluating brightness of a given light when it is utilized with Eq. (2).

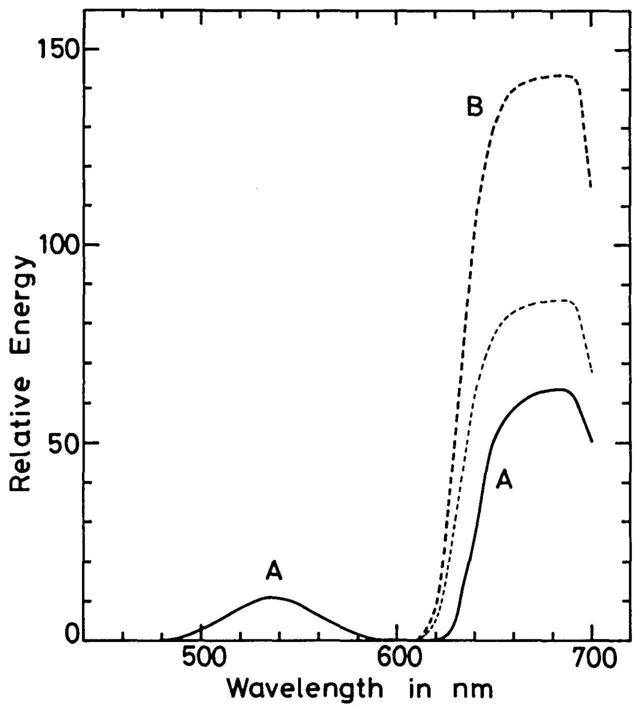

Fig. 7. Spectral-energy distributions of two lights, A and B, that are equated for flicker photometry. When the two lights are equated for brightness by direct comparison, the energy of light B is reduced to that shown by a thin dashed curve.

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